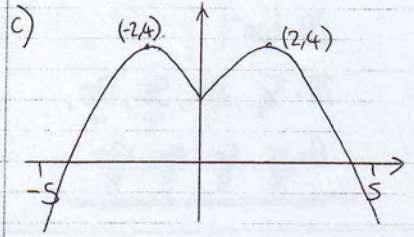
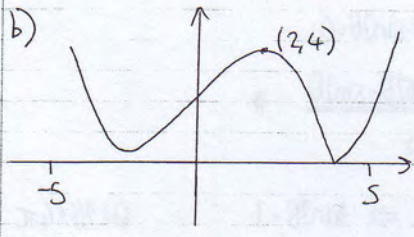
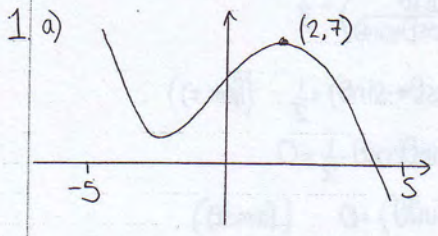


1

Core 3 - Jan 06 Solutions



2

$$\frac{2x^2 + 3x}{(2x+3)(x-2)} - \frac{6}{x^2 - x - 2}$$

$$= \frac{x(2x+3)}{(2x+3)(x-2)} - \frac{6}{(x-2)(x+1)}$$

3

$$\frac{dy}{dx} = \frac{3x \times -6x^2 \sin(2x^3) - 3 \cos(2x^3)}{9x^2}$$

$$= \frac{-6x^3 \sin(2x^3) - \cos(2x^3)}{3x^2}$$

b)  $x = 4 \sin(2y+6)$

$$\frac{dx}{dy} = 8 \cos(2y+6)$$

If  $x = 4 \sin(2y+6)$

$$x^2 = 16 \sin^2(2y+6)$$

$$x^2 = 16 \cos^2(2y+6)$$

$$\cos^2(2y+6) = \frac{1-x^2}{16}$$

$$\cos(2y+6) = \frac{\sqrt{1-x^2}}{4}$$

$$\therefore \frac{dx}{dy} = 8 \frac{\sqrt{1-x^2}}{4}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$$

2

$$= \frac{x(x+1)}{(x+1)(x-2)} - \frac{6}{(x+1)(x-2)}$$

$$= \frac{x^2 + x - 6}{(x+1)(x-2)}$$

$$= \frac{(x-2)(x+3)}{(x+1)(x-2)} = \frac{x+3}{x+1}$$

3  $y = \ln(\frac{1}{3}x)$  when  $x=3$ ,  $y = \ln(\frac{1}{3} \times 3) = \ln(1) = 0$

$$\frac{dy}{dx} = \frac{1}{x} \therefore \frac{dy}{dx} \Big|_{x=3} = \frac{1}{3}$$

$\therefore$  Gradient of normal = -3

When  $x=3$   $y=0 \therefore y-0 = -3(x-3)$

$$y = -3x + 9$$

4 a)  $y = x^2 e^{3x+2}$

$u = x^2 \quad u' = 2x$   
 $v = e^{3x+2} \quad v' = 3e^{3x+2}$

$$\frac{dy}{dx} = 3x^2 e^{3x+2} + 2x e^{3x+2}$$

b)  $y = \frac{\cos(2x^3)}{3x}$

$u = \cos(2x^3) \quad u' = -6x^2 \sin(2x^3)$   
 $v = 3x \quad v' = 3$

4

5 a)  $2x^3 - x - 4 = 0$

$$x^3 = \frac{x+4}{2}$$

$$x^2 = \frac{x+4}{2x} \therefore x^2 = \frac{1}{2} + \frac{2}{x} \quad x = \sqrt{\frac{1}{2} + \frac{2}{x}} \quad \#$$

b)  $x_1 = \sqrt{\frac{2}{1.35} + \frac{1}{2}} = 1.41$

$$x_2 = \sqrt{\frac{2}{1.4077} + \frac{1}{2}} = 1.39$$

$$x_3 = \sqrt{\frac{2}{1.3859} + \frac{1}{2}} = 1.39$$

c)  $f(1.3915) = 2(1.3915)^3 - 1.3915 - 4 = -0.0029$

$$f(1.3925) = 2(1.3925)^3 - 1.3925 - 4 = 0.0078$$

change of sign indicate root and  $\therefore x = 1.392$  to 3dp. #

6 a)  $R \cos(x+\alpha) = 12 \cos x - 4 \sin x$

$$R \cos x \cos \alpha - R \sin x \sin \alpha = 12 \cos x - 4 \sin x$$

$$R \cos \alpha = 12 \quad R \sin \alpha = 4$$

$$\tan \alpha = \frac{4}{12} = \frac{1}{3} \Rightarrow \alpha = 18.4^\circ$$

5

$$R^2 = 12^2 + 4^2 = 160$$

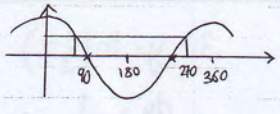
$$\therefore R = 4\sqrt{10}$$

b)  $4\sqrt{10} \cos(x + 18.4^\circ) = 7 \quad 0 \leq x \leq 360$

$$\cos(x + 18.4^\circ) = \frac{7}{4\sqrt{10}} \quad 18.4^\circ \leq x + 18.4^\circ \leq 378.4^\circ$$

$$x + 18.4^\circ = 56.4^\circ, 303.6^\circ$$

$$x = 38.0^\circ, 285.2^\circ$$



c) i) min value  $-4\sqrt{10}$

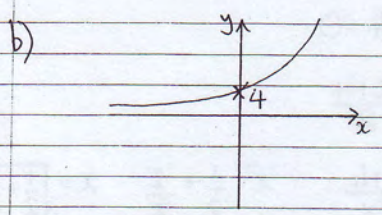
ii) When  $x + 18.4^\circ = 180^\circ \Rightarrow x = 161.6^\circ$

7a) i)  $\frac{\cos 2x}{\cos x + \sin x} = \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x}$

$$= \cos x - \sin x \quad \#$$

ii)  $\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(\cos^2 x - \sin^2 x - 2\sin x \cos x)$   
 $= \frac{1}{2}(\cos^2 x - (1 - \cos^2 x) - 2\sin x \cos x)$   
 $= \frac{1}{2}(2\cos^2 x - 1 - 2\sin x \cos x)$   
 $= \cos^2 x - \frac{1}{2} - \sin x \cos x \quad \#$

7



c)  $gf(x) > 0$

d)  $\frac{d}{dx}(4e^{4x}) = 3$

$$16e^{4x} = 3$$

$$e^{4x} = \frac{3}{16}$$

$$4x = \ln\left(\frac{3}{16}\right)$$

$$x = \frac{1}{4} \ln\left(\frac{3}{16}\right) = -0.418 \text{ (3sf)}$$

6

b)  $\cos \theta \left( \frac{\cos 2\theta}{\cos \theta + \sin \theta} \right) = \frac{1}{2}$

$$\cos \theta (\cos \theta - \sin \theta) = \frac{1}{2} \quad (\text{from i})$$

$$\cos^2 \theta - \sin \theta \cos \theta - \frac{1}{2} = 0$$

$$\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0 \quad (\text{from ii})$$

$$\cos 2\theta - \sin 2\theta = 0$$

$$\cos 2\theta = \sin 2\theta \quad \#$$

c)  $\sin 2\theta = \cos 2\theta$

$$\frac{\sin 2\theta}{\cos 2\theta} = 1 \Rightarrow \tan 2\theta = 1 \quad 0 \leq 2\theta \leq 4\pi$$

$$2\theta = \tan^{-1} 1$$

$$2\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$$

$$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$$

9 a)  $g(f(x)) = e^{2(2x + \ln 2)}$

$$= e^{4x + 2\ln 2}$$

$$= e^{4x} e^{2\ln 2}$$

$$= e^{4x} e^{\ln 4} = \underline{4e^{4x}} \quad \#$$